

**DETERMINING OPTIMAL ALLOCATION AND  
CROP PATTERN IN EGYPT BY THE USE OF  
LINEAR PROGRAMMING**

**LA DETERMINATION DE L'ALLOCATION OPTIMALE  
ET DE L'ASSOLEMENT CULTURAL EN EGYPTE PAR  
L'UTILISATION DE LA PROGRAMMATION LINEAIRE**

By Hesham M. Ali and Mohamed R. Mahmoud

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## **ABOUT THE AUTHORS**

The 4th Dr. Hassan Ismail Memorial International Award, 1999 has been jointly awarded to Mr. Hesham M. Ali and Mr. Mohamed R. Mahmoud for their paper "Determining Optimal Allocation and Crop Pattern in Egypt by the use of Linear Programming" which they jointly presented at the 7th Nile 2002 Conference held in Cairo in March 1999.

Mr. Hesham M. Ali is Researcher, Director of Minister's Office, Ministry of Public Works and Water Resources, Kornish El-Nile, Imbaba-Giza, Egypt.

Mr. Mohamed R. Mahmoud is Researcher, General Supervisor of Information Center and DSS, Ministry of Public Works and Water Resources, Kornish El-Nile Imbaba-Giza, Egypt.

The two authors have been awarded by President, ICID a citation plaque each and prize money of US \$ 1000 shared between them, at the 17th International Congress held at Granada, Spain from 11-19 September 1999.

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LA DETERMINATION DE L'ALLOCATION OPTIMALE  
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**Hesham M. Ali<sup>1</sup> Mohamed R. Mahmoud<sup>2</sup>**

**ABSTRACT**

The Nile River is the main source of water in Egypt. According to the 1959 Sudan treaty, Egypt has a quota of 55.5 billion cubic meters of Nile water per year. Eighty six percent of Egypt's quota of water goes to agriculture, and demand is increasing, due to agricultural expansion and other factors. In addition to the rapidly increasing population, the two great new projects in north Sinai and south valley will demand an annual water volume of more than 9 billion cubic meters per year. Egypt is facing the challenge of increasing water demand by implementing different projects and strategies to conserve water and develop its water resources. Irrigation Improvement Project, Reuse of Drainage Water, and

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1 Researcher, Director of Minister's Office, Ministry of Public Works and Water Resources, Kornish El-Nile, Imbaba-Giza 12666 (Egypt)

2 Researcher, General Supervisor of Information Center and DSS, Ministry of Public Work and Water Resources, Kornish El-Nile, Imbaba-Giza 12666 (Egypt).

Groundwater Development are some of these projects. Reducing the highly water consumptive crop areas is another alternative to conserve water.

Crop consumptive use depends on many factors. Two of these factors are the temperature and humidity which differ in Egypt considerably from north to south. Making use of this formula, this study determines the optimal allocation of the different cultivated crops in Egypt. The actual crop pattern of year 1993 is chosen to be compared with the optimal crop pattern of that year. A linear programming mathematical model is developed. Different alternatives have been investigated where the crop areas are constrained to differ from the actual areas within a certain limits. The amount of irrigation water saved in each alternative is estimated.

### RESUME

Le Nile est la principale source de l'eau en Egypte. D'après le traité de Soudan de 1959, l'Egypte a un quota de 55,5 billion mètres cubes de l'eau du Nile par an. Quatre vingt six pourcent du quota de l'eau de l'Egypte va pour l'agriculture, et la demande est augmentée à cause de l'expansion de l'agriculture et des autres facteurs. En addition à l'augmentation rapide de la population, les deux grands nouveaux projets au Nord du Sinai et au Sud du Vallée va demander un volume d'eau annuel de plus de 9 billion mètres cubes par an.

L'Egypte affronte le défi de l'augmentation de la demande de l'eau par l'exécution de différents projets et les stratégies pour conserver l'eau et développer ses ressources. Le projet de l'amélioration de l'irrigation et la réutilisation de l'eau de drainage, et le développement des eaux souterraines sont parmi ces projets. La réduction des régions des récoltes qui consomment une grande quantité d'eau et une autre alternative pour la conservation de l'eau. L'utilisation de la consommation des récoltes dépend de plusieurs facteurs. Deux de ces facteurs sont la température et l'humidité qui différent en Egypte considérablement du Nord au Sud. En utilisant ce formula, cette étude détermine cette optimale allocation de différents récoltes cultivées en Egypte. Le système optimal des récoltes de cet an. Un model mathématique de programmation linéaire est développé. De différentes alternatives ont été examinées car les régions de récoltes sont contraintes d'être différentes de régions actuelles par certaines limites. La quantité de l'eau de l'irrigation sauvée dans chaque alternative a été estimée.

### INTRODUCTION

Water management in irrigated agricultural areas is the most important way to get the optimal crop yield with the optimal use of the available water resources. The two main limits for increased agricultural production are the availability of

additional cultivable land and the adequate irrigation water. Agriculture consumes about 85% of Egypt water resources.

Due to shortage of irrigation water, the optimal allocation of crop area in irrigated agriculture could be considered. Crop consumptive use depends on many factors. Two of these factors are the temperature and the humidity, which differ considerably from north to south Egypt. Taking into consideration this folumina, the optimal allocation and crop pattern of the different cultivated crops in Egypt have been determined. Seven cases have been studied where the crop areas have been allowed to be changed from the actual areas.

A linear programming mathematical model is developed and the amount of irrigation water is estimated in each case. Development of mathematical models to generate optimal irrigation policies has been performed by researchers since 1970. Most of optimization models adopted linear programming (LP) and dynamic programming (DP). The use of LP is limited that the object function and constraints are linear.

### MODEL FORMULATION

The object function of the crop area allocation model is to minimize the amount of irrigation water used. As the object function and the constraints are of the linear form, a linear programming model can be used. The linear programming technique is utilized to determine the optimal allocation of the cultivated crops taking into consideration the specified constraints. The Egyptian agricultural land is divided into three main regions : Upper (u), Middle (m), and lower (l). Each region is characterized with its climate and consequently its crop consumptive use.

#### Objective Function

$$\begin{aligned} \text{Min } z = & \frac{1}{E_1} \left( \sum_{i=1}^{C_w} C_d \times W_{il} \right) + \frac{1}{E_m} \left( \sum_{i=1}^{C_w} C_{im} \times W_{im} \right) + \frac{1}{E_u} \left( \sum_{i=1}^{C_w} C_{iu} \times W_{iu} \right) + \frac{1}{E_1} \left( \sum_{i=C_w+1}^{C_w+C_s} C_d \times W_{il} \right) \\ & + \frac{1}{E_m} \left( \sum_{i=C_w+1}^{C_w+C_s} C_{im} \times W_{im} \right) + \frac{1}{E_u} \left( \sum_{i=C_w+1}^{C_w+C_s} C_{im} \times W_{im} \right) \end{aligned}$$

Where :

- Z = Irrigation water volume
- $C_w$  = No. of winter crops
- $C_s$  = No. of summer nili crops
- $E_i$  = Lower Egypt irrigation efficiency

- $E_m$  = Middle Egypt irrigation efficiency  
 $E_u$  = Upper Egypt irrigation efficiency  
 $C_{il}$  = Area of crop No.  $i$  Cultivated in Lower Egypt  
 $C_{im}$  = Area of crop No.  $i$  cultivated in Middle Egypt  
 $C_{iu}$  = Area of crop No.  $i$  cultivated in Upper Egypt  
 $W_{il}$  = Water consumptive use of crop  $i$  cultivated in Lower Egypt  
 $W_{im}$  = Water consumptive use of crop  $i$  cultivated in Middle Egypt  
 $W_{iu}$  = Water consumptive use of crop  $i$  cultivated in Upper Egypt

### Constraints

The object function is subject to the following constraints :

$$\sum_{i=1}^{C_w} C_{il} = A_{l1}$$

$$\sum_{i=C_w+1}^{C_w+C_s} C_{il} = A_{l2}$$

$$\sum_{i=1}^{C_w} C_{im} = A_{m1}$$

$$\sum_{i=C_w+1}^{C_w+C_s} C_{im} = A_{m2}$$

$$\sum_{i=1}^{C_w} C_{iu} = A_{u1}$$

$$\sum_{i=C_w+1}^{C_w+C_s} C_{iu} = A_{u2}$$

$$C_{il} \times A_i \leq C_{il} + C_{im} + C_{iu} \leq C_{i2} \times A_i,$$

$$i = 1 \rightarrow C_w + C_s$$

Where :

$A_{l1}$ ,  $A_{m1}$ , and  $A_{u1}$  are the available winter agricultural areas in lower, middle, and Upper Egypt while  $A_{l2}$ ,  $A_{m2}$  and  $A_{u2}$  are the available Summer and Nili agricultural areas

$$\begin{aligned}
C_{il} \times A_i &= \text{minimum allowable area of crop } i \\
C_{i2} \times A_i &= \text{maximum allowable area of crop } i \\
A_i &= \text{Total actual crop } i \text{ area}
\end{aligned}$$

### MODEL APPLICATION

The model is used to determine the optimal allocation and the crop pattern of the different crops in Egypt. The actual crop pattern of the year 1993 is used to be compared with the optimal crop pattern. The total cultivated area and the cultivated areas of lower, middle, and Upper Egypt are kept without change. The areas of the different crops within the three regions will be allowed to change so that the optimal allocation of these crops is determined. Different cases will be studied where the total area of each crop is allowed to change within a certain limits. These limits include : 100%, 95% - 105%, 90% - 110%, 80% - 120%, 70% - 130%, 60% - 140%, 50% - 150%.

The optimal allocation of the crop areas and amount of water volume saved in each case will be determined. Table (1) shows the crop consumptive use in lower, middle, and upper Egypt while Table (2) shows the actual crop pattern in year 1993. Winter crops are the crops from No. 1 to 14 while Summer and Nili crops are the others.

Assuming the same irrigation water efficiency in lower, middle, and upper Egypt, the object function can be written as :

$$\begin{aligned}
\text{Min } Z &= \sum_{i=1}^{14} C_{il} \times W_{il} + \sum_{i=1}^{14} C_{im} \times W_{im} + \sum_{i=1}^{14} C_{iu} \times W_{iu} + \\
&\sum_{i=15}^{28} C_{il} \times W_{il} + \sum_{i=15}^{28} C_{im} \times W_{im} + \sum_{i=15}^{28} C_{iu} \times W_{iu}
\end{aligned}$$

Subject to :

$$\sum_{i=1}^{14} C_{il} = A_{l1}$$

$$\sum_{i=15}^{28} C_{il} = A_{l2}$$

$$\sum_{i=1}^{14} C_{im} = A_{m1}$$

$$\sum_{i=15}^{28} C_{im} = A_{m2}$$

$$\sum_{i=1}^{14} C_{iu} = A_{u1}$$

$$\sum_{i=15}^{28} C_{iu} = A_{u2}$$

**Table 1. Crop Consumptive Use**

| Crop No. | Crop Name           | Crop Consumptive Use (m <sup>3</sup> /feddan) |              |             |
|----------|---------------------|---|--------------|-------------|
|          |                     | Lower Egypt                                   | Middle Egypt | Upper Egypt |
| 1.       | Wheat               | 1608.6  | 1996.6       | 2195.3      |
| 2.       | Horse Beans         | 1281  | 1588.8       | 1827        |
| 3.       | Barly               | 1408  | 1800         | 2154        |
| 4.       | Fenugreek           | 1000  | 1080         | 1242        |
| 5.       | Iupine              | 980   | 1080         | 1242        |
| 6.       | Check peas          | 1012  | 1105         | 1270.8      |
| 7.       | Lentiles            | 1335  | 1503.6       | 1617        |
| 8.       | Clover, short       | 877.8   | 1092         | 1188.6      |
| 9.       | Clover, full        | 2364.6  | 2842.2       | 3120.7      |
| 10.      | Flax                | 1407  | 1512         | 1550        |
| 11.      | Onion, winter       | 1629.4  | 1722         | 1722        |
| 12.      | Garlic              | 1360.8  | 1608.6       | 7608.6      |
| 13.      | Winter veg.         | 1360.8  | 1608.6       | 1608.6      |
| 14.      | Other winter plants | 1113  | 1218         | 1218        |
| 15.      | Cotton              | 2818.2  | 3541.5       | 3881.2      |
| 16.      | Rice                | 4691.4  | 4691.4       | 5395.1      |
| 17.      | Maize, summer       | 2430.2  | 2612.4       | 2805.6      |
| 18.      | Sorgum, summer      | 2338  | 2545.2       | 2751        |
| 19.      | Say beans           | 2020.2  | 2587.2       | 2975.4      |
| 20.      | Sesame              | 2047.6  | 2255         | 2593.3      |
| 21.      | Groundnuts          | 3399  | 3680         | 4232.2      |
| 22.      | Summer veg.         | 1925.2  | 2343.6       | 2343.6      |
| 23.      | Other summer plant  | 2070.6  | 2402.4       | 2625        |
| 24.      | Maize, Nili         | 2251.2  | 2310         | 2370        |
| 25.      | Sorgum, Nili        | 2178.6  | 2256.8       | 2340        |
| 26.      | Nile vegetables     | 1541.4  | 1604.4       | 1670        |
| 27.      | Gardents            | 4014.8  | 4141         | 5418        |
| 28.      | Sugarcane           | —   | 7167.8       | 9190.6      |



**Table 2.** Actual Crop Pattern in Year 1993

| Crop No. | Area (Feddan) |              |             |
|----------|---------------|--------------|-------------|
|          | Lower Egypt   | Middle Egypt | Upper Egypt |
| 1.       | 1051853       | 382582       | 394797      |
| 2.       | 102095        | 48988        | 50295       |
| 3.       | 44831         | 11717        | 5453        |
| 4.       | 970           | 10789        | 3207        |
| 5.       | 3475          | 2002         | 1989        |
| 6.       | 1216          | 595          | 18537       |
| 7.       | 6489          | 355          | 10697       |
| 8.       | 586346        | 10382        | 45915       |
| 9.       | 1160295       | 340524       | 168027      |
| 10.      | 28553         | 369          | 0           |
| 11.      | 12941         | 11588        | 4554        |
| 12.      | 3519          | 13675        | 1421        |
| 13.      | 284362        | 88239        | 87741       |
| 14.      | 3110          | 31322        | 12075       |
| 15.      | 660915        | 158615       | 64780       |
| 16.      | 1254452       | 21843        | 0           |
| 17.      | 946658        | 421965       | 226908      |
| 18.      | 0             | 48921        | 284607      |
| 19.      | 9151          | 28048        | 6095        |
| 20.      | 11418         | 14697        | 28747       |
| 21.      | 15567         | 10184        | 5518        |
| 22.      | 285009        | 104885       | 28980       |
| 23.      | 86587         | 43679        | 26415       |
| 24.      | 113471        | 156138       | 41336       |
| 25.      | 0             | 11571        | 468         |
| 26.      | 61122         | 65496        | 13057       |
| 27.      | 382207        | 91346        | 46722       |
| 28.      | 0             | 37249        | 236508      |
| Total    | 7116612       | 2260944      | 1814846     |

$$C_{il} \times A_i \leq C_{il} + C_{im} + C_{iu} \leq C_{i2} \times A_i$$

$$i = 1 \rightarrow 28$$

$$C_{28i} = 0$$

Where :

$$A_{i1} = 3290055 \text{ feddan}$$

$$A_{i2} = 3826557 \text{ feddan}$$

$$\begin{aligned}
 A_{m1} &= 1046307 \text{ feddan} \\
 A_{m2} &= 1214637 \text{ feddan} \\
 A_{u1} &= 804708 \text{ feddan} \\
 A_{u2} &= 1010141 \text{ feddan}
 \end{aligned}$$

### RESULT ANALYSIS

The linear programming Model is solved to determine the optimal allocation of the 28 crops in lower, middle, and upper Egypt. Seven different cases have been studied where both the minimum and maximum allowed total crop area is changed from the actual crop area cultivated in the year 1993 these cases are :

1.  $C_{i1}, C_{i2} = 100\%, 100\%$
2.  $C_{i1}, C_{i2} = 95\%, 105\%$
3.  $C_{i1}, C_{i2} = 90\%, 110\%$
4.  $C_{i1}, C_{i2} = 80\%, 120\%$
5.  $C_{i1}, C_{i2} = 70\%, 130\%$
6.  $C_{i1}, C_{i2} = 60\%, 140\%$
7.  $C_{i1}, C_{i2} = 50\%, 150\%$

Figures 1 - 3 show the actual and the optimal crop allocation in lower, middle, and upper Egypt for the seven different cases. It is clear from these figures that the allocation and the distribution of all the crops have been changed from the actual case. The cultivated area of these crops with high consumptive use has been decreased while those with low consumptive use have been increased.

The main objective of this study is to minimize the irrigation water used through the optimal allocation of the crop area. The total water volume saved in the seven cases have been estimated. Assuming irrigation efficiency 0.65, Figure 4 shows the total water volume saved in the seven cases. It is clear that the total water volume saved ranges between 0.974 billion  $m^3$  and 8.185 billion  $m^3$ .

### CONCLUSION AND RECOMMENDATION

A linear programming mathematical model has been developed to determine the optimal allocation and the crop pattern of an agricultural area to minimize the amount of the irrigation water used. The model is applied on the agricultural area of Egypt considering that it is divided into three regions; lower, middle, and upper Egypt. The optimal crop area allocation is determined for the year 1993. Seven different cases have been studied where the crop area has been allowed to change within a certain limits.

The results show that a considerable amount of water volume can be saved. The saving is increased by increasing the limits within which the crop area is

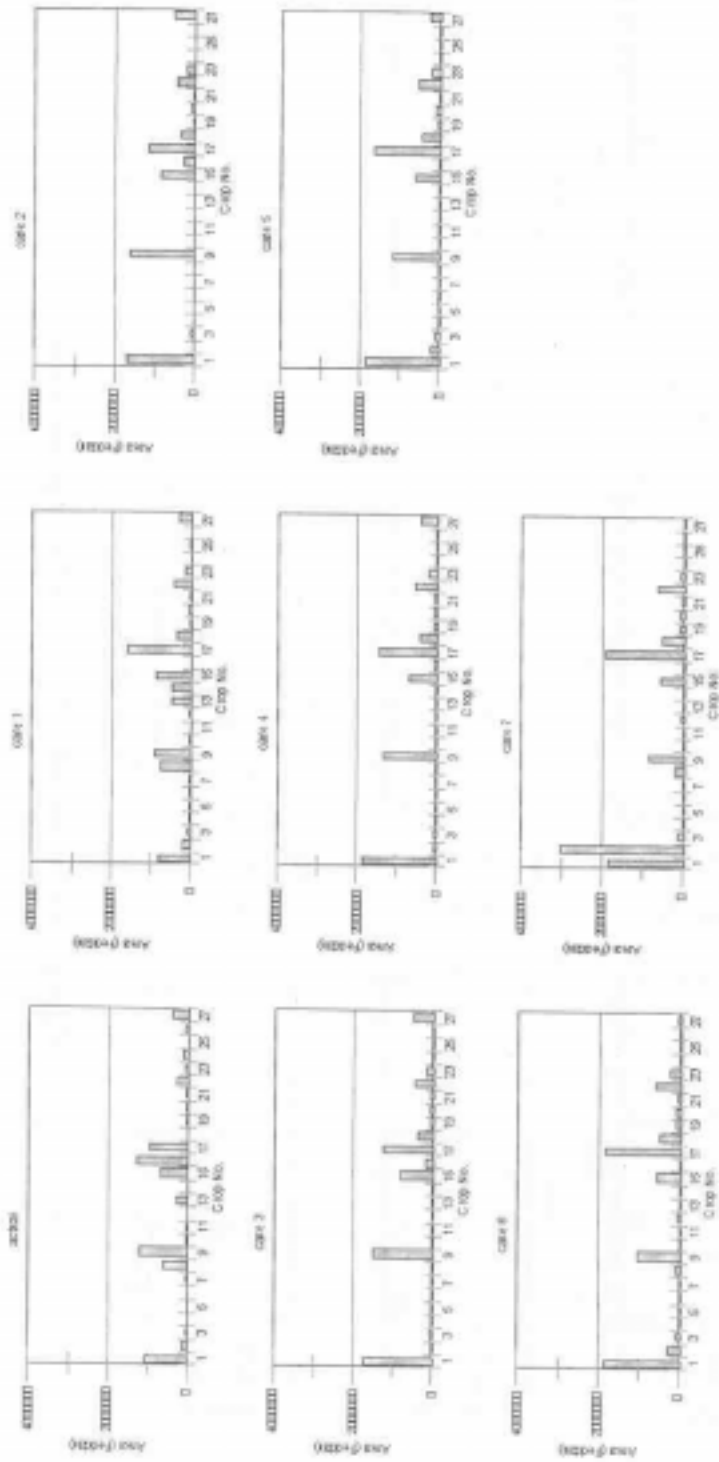


Figure 1. Actual and optimal crop allocation (Lower Egypt)

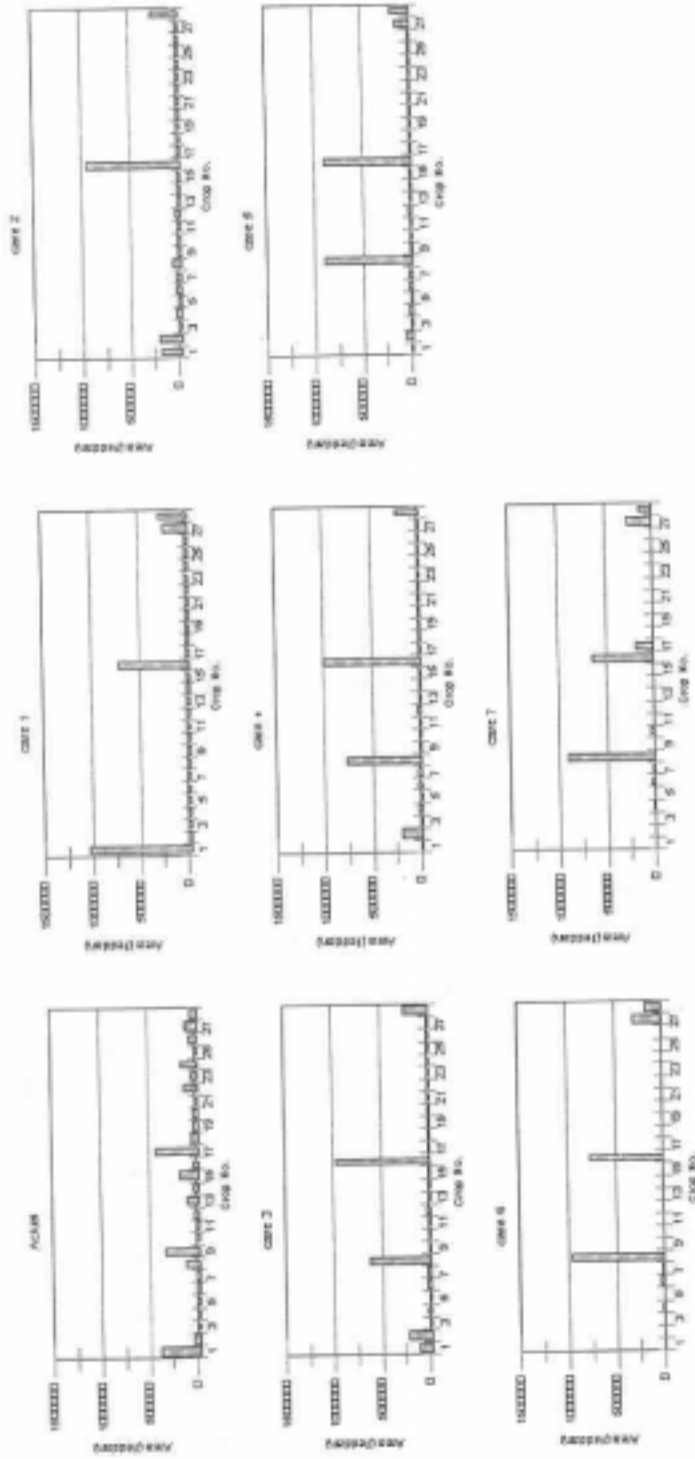


Figure 2. Actual and optimal crop allocation (Middle Egypt)

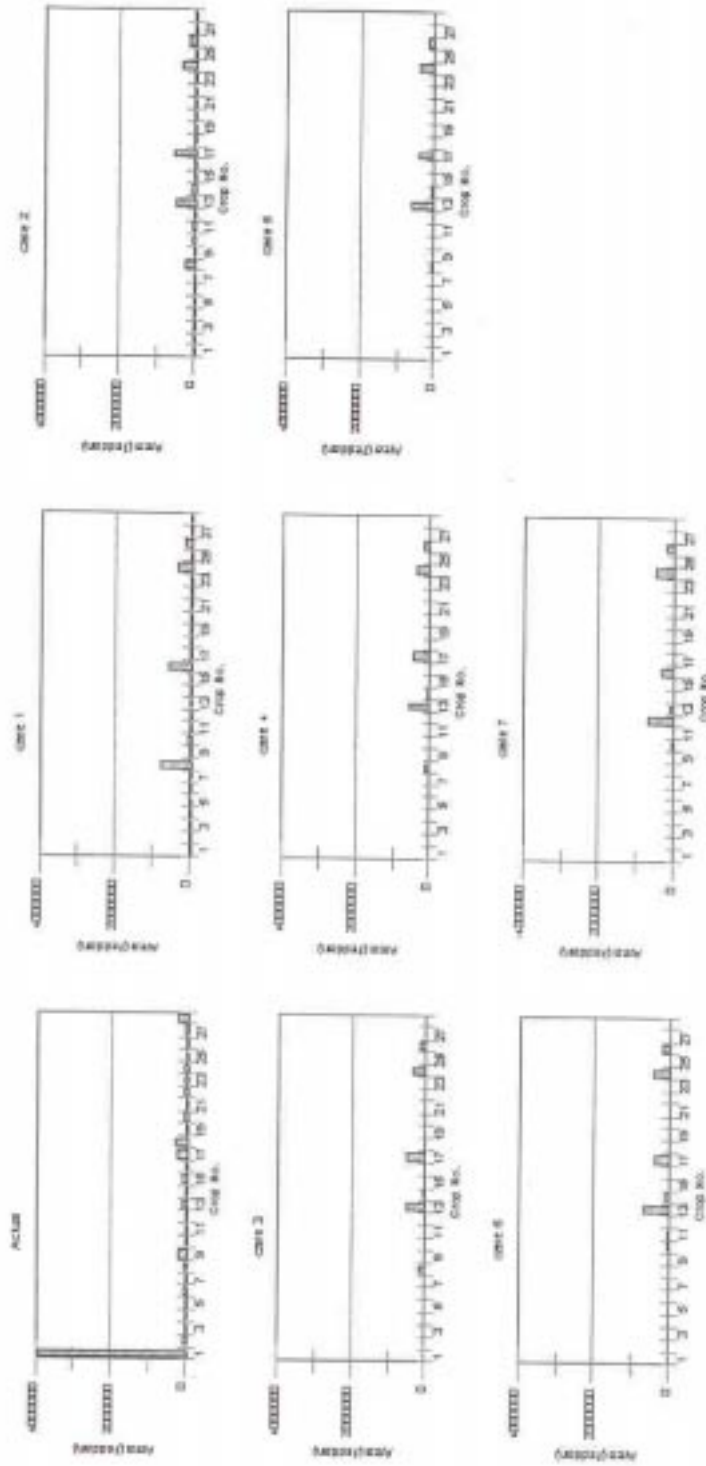
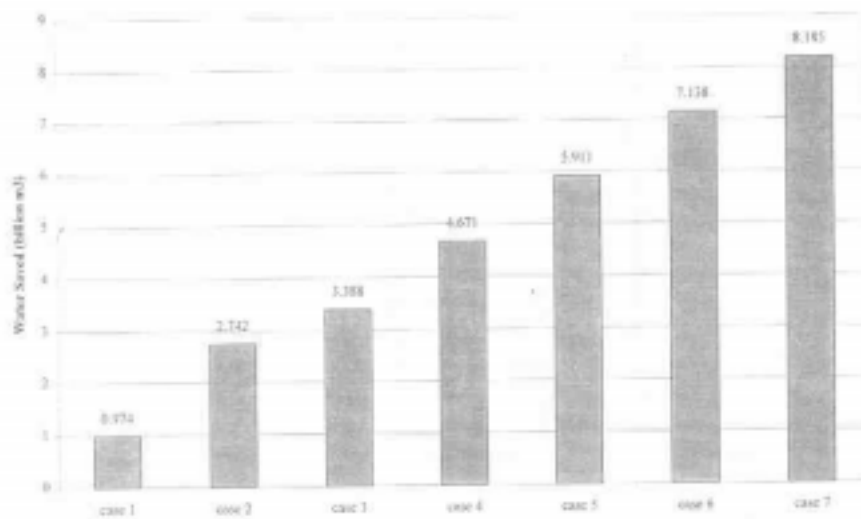


Figure 3. Actual and optimal crop allocation (Upper Egypt)



**Figure 4.** Water volume saved in the seven different cases

allowed to change. The water volume saved is estimated to be about 8.185 billion  $m^3$  when the crop area are allowed to change between 0.5 and 1.5 the actual cultivated crop area. This study helps to cultivate the total agricultural area in Egypt when there is a shortage of the irrigation water.

It is recommended that a further study to be conducted to investigate the economic of applying this model and a study to be conducted on another different years.

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### **Dr. Hassan M. Ismail**

Late Dr. Hassan M. Ismail was a highly respected national and international personality. He was Professor of Hydraulics and Dean of Faculty of Engineering, Cairo University. Later he became the President of Cairo University, President of the Academy of Science, Egypt and Minister of Higher Education. He was also the President of the leading consultancy firm PACER and member of several international organizations, associations and societies. Dr. Ismail Hassan dedicated his entire life to the development of science and to educating generations of irrigation and drainage engineers in Egypt. He firmly believed in the role of ICID in ensuring international cooperation in the advancement of the world knowledge and experience in irrigation and drainage. Dr. Ismail had a great faith in young professionals, especially from Africa to carry out creative work in water management and sustainable development in irrigated agriculture.



Dr. Hassan Ismail was the Chairman of the Egyptian National Committee on Irrigation and Drainage for about 16 years.

*"The Dr. Hassan M. Ismail Memorial International Award" has been established with effect from the 16th ICID Congress at Cairo in 1996 to be awarded to a young professional for the best paper on the development of irrigation, drainage and flood control in Africa, submitted to an ICID Congress or ICID's Afro-Asian Regional Conference or an African Conference.*

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**INTERNATIONAL COMMISSION ON IRRIGATION AND DRAINAGE**

48 Nyaya Marg, Chanakyapuri, New Delhi 110021, India

Tel : 611 5679, 611 6837, 467 9532; Fax : 91-11-611 5962

E-Mails : [icbiad@glasd101.vsnl.net.in](mailto:icbiad@glasd101.vsnl.net.in); [icid@vsnl.com](mailto:icid@vsnl.com)

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